Robust Normative Comparisons of Socially Risky Situations*

Nicolas Gravel[†] and Benoît Tarroux[‡]

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Abstract

In this paper, we characterize and empirically implement robust normative criteria for comparing societies on the basis of their allocations of *risks* among their members. Risks are modelled as lotteries on the set of distributions of state-contingent pecuniary consequences. Individuals are assumed to have individualistic Von Neuman-Morgenstern preferences for these risks. Appealing to Harsanyi's aggregation theorem, we provide empirically implementable criteria that coincide with the unanimity, over all such individual preferences, of anonymous and Pareto-inclusive Von Neuman Morgenstern social rankings of risks. The empirically implementable criteria can be interpreted as *sequential expected poverty dominance*. Illustrations of the usefulness of the criteria for comparing the exposure to unemployment risk of different segments of the French and US workforce and for appraising the evolution, over time, of risks of violent crimes in India are also provided.

Keywords: Risk, Dominance, ex-ante Social Welfare,Expected Poverty, Unemployment, Crime.

JEL classification numbers: C81, D3, D63, D81, I32, J63, J64

1 Introduction

The exposure to various kinds of *risks* that societies provide to their members is a clearly important ingredient for normative evaluation. For instance, some countries, like US or UK, are commonly depicted as having "flexible"

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†Université de la Méditerrannée & IDEP-GREQAM, Centre de la Vieille Charité, 2, rue de la Charité, 13 002 Marseille, France nicolas.gravel@univmed.fr.

‡Université Charles de Gaulle (Lille 3) & IDEP, Domaine universitaire Pont-du-Bois, rue du Barreau, BP 60149, 59 653 Villeneuve d'Ascq, Cedex, France, tarrouxb@hotmail.fr.

labour markets in which most of the work force faces a mild risk of unemployment with little compensation and where the wages of those employed are relatively high. Other countries, like France, are to the contrary portrayed as having "rigid" labour markets in which a fraction of the work force is fully protected against the risk of being unemployed, even though it enjoys moderate wages, while the remaining part of the work force is exposed to a high risk of unemployment which, if it arises, is the object of significant pecuniary compensation. A natural question to ask from a normative point of view is: what form of organization of the labour market is better? Analogously, one may be interested in comparing different countries - or the same country at different points in time - on the basis of their distributions of income and exposure to crime, or to risks of health (see e.g. Gravel *et al.* (2008) or Gravel and Mukhopadhyay (2007)).

In this paper, we theoretically characterize and empirically implement *robust* criteria for comparing societies in terms of their performance in allocating risks between their members. The risks to which the analysis applies are described by finite lists of probabilities of occurrence of states of nature (being unemployed, being employed, etc.) and of pecuniary consequences contingent on those states. Moreover, we also allow risks to have non-pecuniary consequences in the sense that a given amount of money may be valued differently according to the states in which it is received. When risks have non-pecuniary consequences in this sense, we assume that states of nature can be unambiguously ordered from the worst (e.g. being gravely ill) to the best (e.g. being in perfect health).

Assuming that individuals have Von Neuman-Mongenstern (VNM) preferences over risks, and acknowledging that a *distribution* of these *individual risks* can be seen as a *socially risky situation*, we derive empirically implementable criteria for comparing socially risky situations that *coincide* with the unanimity, over some (reasonably large) class of individual VNM preferences, of all Pareto consistent social rankings that satisfy themselves the VNM properties. It is in this (unanimity) sense that the criteria are considered as "robust". Because of Harsanyi (1955)'s aggregation theorem, we know that a VNM ranking of socially risky situations that respects, in the usual Pareto-sense, individual VNM preferences *can be* thought of as resulting from the comparisons of the *sum* of the individuals' VNM expected utility functions. Hence, a ranking of socially risky situations agreed upon by all social VNM rankings who respect individuals' VNM rankings is nothing else than a ranking that commands unanimity over all sums of individual VNM expected utility functions taken in some suitable class.

Normative comparisons of distributions of socially risky situations can be seen as particular instances of *multi-dimensional* normative evaluation (see e.g. Atkinson and Bourguignon (1982), Kolm (1977)) in which one is interested in comparing distributions of several attributes by requiring the unanimity over a class of utilitarian rankings. Yet, the additional structure imposed by the fact that the considered attributes are ingredients of risks evaluated, both at the social and the individual level, by VNM preferences turns out to be quite significant. As shall be seen, the empirically implementable criteria that are obtained in this context are quite different from the abstract first or second order multidimensional stochastic dominance criteria of the Atkinson and Bourguignon (1982) variety, even though they bear a formal similarity with *two-dimensional* dominance criteria proposed by Atkinson and Bourguignon (1987) and generalized by Jenkins and Lambert (1993) and Bazen and Moyes (2003).

We characterize specifically *two* implementable criteria, each of which corresponding to a specific property of the VNM individual utility functions. We first consider VNM utility functions whose marginal utility of income is positive and weakly decreasing with respect to the states (ordered from the worst to the best). We find that the empirically implementable criterion that corresponds to the unanimity over this class of VNM utilities is what we call "*Sequential Expected Headcount Poverty*" (SEHP) dominance. According to this criterion, socially risky situation A is better than situation B if, for any poverty line and any state, the *expected number of individuals* who are below the line *and* in a worse state is no greater in A than in B. The sequential aspect of the criterion arises from the fact that in order to check for dominance, one looks first at expected number of poor in the worst state and, in a second step, at the total expected number of poor in the two worst states, and so on, in a sequential fashion. To that extent, this criterion may be viewed as giving a *priority to poverty* that is decreasing with states. This reflects of course the assumption that the marginal utility of income is decreasing with respect to states.

The second, and more restricted, family of VNM utility functions considered satisfy, in addition to the above properties, the requirement that the marginal utility of income is decreasing with income in every state at a rate that is decreasing with the state. We then show that the implementable criterion that coincides with the unanimity over this class of VNM utility functions, of all Pareto-inclusive VNM social preferences is what we call "*Sequential Expected Poverty Gap*" (SEPG) dominance. This criterion works just like the SEHP one, but with poverty gap, rather than headcount poverty, used as the poverty measure. The SEPG criterion is formally analogous to Jenkins and Lambert (1993) dominance criterion - and in fact formally identical to Bazen and Moyes (2003)) criterion - for comparing income distributions between households differing in needs. The analogy is readily seen by interpreting states of nature (ordered from the worst to the best) as need categories and by viewing the ratio of the sum of individual probabilities of falling in any state over the number of individuals as the marginal distribution of needs in the population.

While these two criteria are characterized in a general setting where risks can have both pecuniary and non-pecuniary consequences, it is easy to provide similar, and more discriminatory, dominance criteria for the case where risks have only pecuniary consequences. In such a setting, the VNM utility function is assumed to be the same in all states of natures and the criteria characterized happen to be the *Expected Headcount Poverty* (EHP) and *Expected Poverty Gap* (EPD) dominance. These criteria work like their "sequential" counterpart, but with the important difference that expected poverty (be it headcount or poverty gap) is calculated non-sequentially in all states, rather than sequentially starting from the worst state. Since it is easier to obtain poverty dominance globally than sequentially for every state, the two expected poverty criteria are more discriminatory than their sequential counterparts. This gain in discriminatory power comes, however, at the cost of assuming that risks have only pecuniary consequences.

That these criteria are helpful for comparing societies is illustrated with data on unemployment risk in France and US and on crime risk in India. In the later case, we show that the SEHP criterion is more discriminatory than the corresponding abstract Atkinson and Bourguignon criterion. In the former case, we show that our criteria do not enable one to compare US and France in terms of their allocation of unemployment risks. The empirical illustration also reveals that, in France, male adults are better protected against the risk of unemployment than female ones but, in the US, such a dominance of males over females does not hold. This suggests therefore that the male-female gap in terms of protection against unemployment risks is somewhat higher in France than in the US. The analysis also reveals that young segments of the workforce have worse exposure to unemployment risks than older ones, but that this advantage of the old over the young is somewhat lower in the US than in France.

The plan of the remaining of the paper is as follows. The next section introduces the normative and empirically implementable criteria and establishes the formal equivalence between them (leaving the proofs in the appendix). The third section applies the criteria to the US-France comparisons of labour market risk and to the evaluation of the evolution, over time, of the distribution of individual consumption and crime in India. The fourth section concludes.

2 Theory

2.1 Normative criteria

We consider societies made of a *given* number, n say, of individuals[1], indexed by i, with $i \in N = \{1, ..., n\}$. Societies expose their members to risks in which every individual can fall into a finite number, l say, of mutually

[1] The generalization of the results to societies involving different number of individuals is immediate.

exclusive *states of nature* indexed by j, with $j \in S = \{1, ..., l\}$. We consider in turn two settings in which these states of nature can be appraised. First, we admit the possibility for individuals to attach *intrinsic value* to the state in which they fall (as they may, for instance, intrinsically prefer being healthy than being ill). We do that by assuming that states are ordered from the worst to the best so that state j is weakly worse than state $j+1$ for every $j = 1, ..., l-1$. While individuals may value states intrinsically, they also value them for the pecuniary consequences that arise if they occur. Specifically, in each state in which an individual can fall, he or she receives a non-negative[2] *pecuniary consequence* which we refer to as "income". Later on, we shall consider the more restricted *state independant* case where individuals do not value states intrinsically but care only about the state-contingent pecuniary consequences.

We call *socially risky situation* a specific pattern of exposure of individuals to risks. Formally, we model such a socially risky situation as a finite probability distribution, or a *lottery*, p on the set $\mathbb{X} = (S \times \mathbb{R}_+)^n$ of all vectors of state-income pairs, one such pair for every individual. A typical element x of $\mathbb{X}$ writes:

$$x = (s_1, y_1, ...s_n, y_n)$$

where, for $i = 1, ..., n$, $s_i \in S$ denote the state in which i falls and y_i denote i's income in that state. Hence $p(s_1, y_1, ...s_n, y_n)$ is the (joint) probability that individual i (for $i = 1, ..., n$) falls in the state s_i and gets income y_i. We also denote by $(s_i, y_i; s_{-i}, y_{-i})$ the vector of state-income pairs where individual i gets the pair $(s_i, y_i) \in S \times \mathbb{R}_+$ and all individuals other than i get the vector of state-income pairs $(s_{-i}, y_{-i}) \in (S \times \mathbb{R}_+)^{n-1}$. To be consistent with our interpretation of lotteries as socially risky situations, we require them to satisfy the following condition:

Condition 1 *For every individual i, if $p(s_i, y_i; s_{-i}, y_{-i}) > 0$ for some $(s_i, y_i) \in S \times \mathbb{R}_+$ and $(s_{-i}, y_{-i}) \in (S \times \mathbb{R}_+)^{n-1}$, then $p(s_i, y_i'; s_{-i}', y_{-i}') = 0$ for all $y_i' \in \mathbb{R}_+$ such that $y_i' \neq y_i$ and all $(s_{-i}', y_{-i}') \in (S \times \mathbb{R}_+)^{n-1}$.*

In words, this condition requires lotteries to *never* assign positive probability to two different incomes received by an individual in a given state, even if the two different incomes are obtained for different states and/or incomes for the others. Notice that this condition implies that lotteries have a finite support. It also implies a certain form of independence between individual risks (the income received by someone can not depend upon the state of others). Yet the condition is weaker than that of statistical independence of individual risks. For instance, a socially risky situation in which either everybody suffers from a disease (and gets an income contingent on this state) or nobody does (and gets some other income as a result) satisfies the

[2] The assumption that pecuniary consequences are non-negative is inessential.

condition, even though the individual risks described by this example are not independent. Let $\mathbb{L}$ be the set of all lotteries on $\mathbb{X}$ that satisfy condition 1. Since a lottery p in $\mathbb{L}$ assigns a positive probability to a unique income for individual i in every state $j \in S$ in which i can fall with positive probability, we denote by y_{ij}^p the value of this unique income. Moreover, since, for every individual and state achieved with positive probability, there is a unique income received by the individual in that state, we can think of a lottery p in $\mathbb{L}$ as inducing a lottery π^p on the set S^n of combinations of individual states, and we can accordingly denote by $\pi^p(s)$ the (joint) probability of the individuals being in the configuration of states $s = (s_1, ..., s_n)$. Formally, $\pi(s) = p(s_1, y_{1s_1}^p, ..., s_n, y_{ns_n}^p)$.

Every individual i is assumed to have a VNM preference ordering[3] $\succsim_i$ on $\mathbb{L}$, with asymmetric and symmetric factors $\succ_i$ and $\sim_i$ respectively. This means that there exists a utility function $\Phi_i : (S \times \mathbb{R}_+)^n \to \mathbb{R}$ such that, for every socially risky situations p and q in $\mathbb{L}$, one has:

$$p \succsim_i q \Leftrightarrow \sum_{s \in S^n} \pi^p(s)\Phi_i(s_1, y_{1s_1}^p, ..., s_n, y_{ns_n}^p) \geq \sum_{s \in S^n} \pi^q(s)\Phi_i(s_1, y_{1s_1}^q, ..., s_n, y_{ns_n}^q) \tag{1}$$

We refer to the numerical representation of $\succsim_i$ provided by (1) as to the expected utility representation. We further assume that each individual is *selfish* and, therefore, only cares about the state in which he or she falls and the pecuniary consequence he or she gets in that state. We suppose also that individuals have the same selfish preference. In order to write formally this condition, we notice that any lottery p in $\mathbb{L}$ induces, through the lottery π^p, an individual i's (marginal) lottery π_i^p on S for every individual $i \in N$. We denote by π_{ij}^p this marginal probability that individual i falls in the state j (for $j \in S$) if the socially risky situation is p. This marginal probability π_{ij}^p is defined by:

$$\pi_{ij}^p = \sum_{\{s \in S^n : s_i = j\}} \pi^p(s).$$

With this piece of notation, the assumption that individuals have the same selfish VNM preference means that, for every state $j \in S$, there exists a function $U_j : \mathbb{R}_+ \to \mathbb{R}$ such that, for every i and every socially risky situations p and q in $\mathbb{L}$, one has:

$$p \succsim_i q \Leftrightarrow \sum_{j \in S} \pi_{ij}^p U_j(y_{ij}^p) \geq \sum_{j \in S} \pi_{ij}^p U_j(y_{ij}^q). \tag{2}$$

Socially risky situations in $\mathbb{L}$ are compared by a *social ordering* $\succsim$ (with asymmetric and symmetric factors $\succ$ and $\sim$ respectively) that satisfies the VNM properties and the weak Pareto principle with respect to individual preferences. In the same fashion as in (1), the first property means that

[3] An ordering is a reflexive, complete and transitive binary relation.

there exists a function $\Phi : (S \times \mathbb{R}_+)^n \to \mathbb{R}$ such that, for every socially risky situations p and q in $\mathbb{L}$, one has:

$$p \succsim q \Leftrightarrow \sum_{s \in S^n} p(s)\Phi(s_1, y^p_{1s_1}, ..., s_n, y^p_{ns_n}) \geq \sum_{s \in S^n} q(s)\Phi(s_1, y^q_{1s_1}, ..., s_n, y^q_{ns_n}) \tag{3}$$

The second property requires $\succsim$ to be such that, for two socially risky situations p and q in $\mathbb{L}$, if $p \succ_i q$ for all i, then $p \succ q$.

By virtue of a version of Harsanyi (1955)'s aggregation theorem due to Weymark (1993), any VNM social ordering of $\mathbb{L}$ that respects the weak Pareto principle can be written as a (positively) weighted sum of the individual's expected utility representations of their VNM preference. We formally state this fact as follows.

Proposition 1 *Let $(\succsim_1, ..., \succsim_n)$ be a profile of identical selfish VNM individual preference orderings on $\mathbb{L}$ and let $\succsim$ be a VNM social ordering of $\mathbb{L}$ that satisfies the weak Pareto principle. Then, there exists non-negative numbers $\lambda_1, ...\lambda_n$, one of which at least being strictly positive, such that, for every two lotteries, p and q in $\mathbb{L}$, one has:*

$$p \succsim q \Leftrightarrow \sum_{i \in N} \lambda_i \sum_{j \in S} \pi^p_{ij} U_j(y^p_{ij}) \geq \sum_{i \in N} \lambda_i \sum_{j \in L} \pi^q_{ij} U_j(y^q_{ij}) \tag{4}$$

Proof. *The result follows immediately from proposition 4 in Weymark (1993), after noticing that the condition called "independent prospects" in Weymark (1993) is satisfied herein if $(\succsim_1, ..., \succsim_n)$ are identical selfish VNM individual preferences, and after applying definition (1) to individual preferences.* ■

If one further assumes that the social ranking is *anonymous* in the sense of being indifferent to a permutation of the individuals' names,[4] then the λ_is of condition (4) must all be identical. Because of this, it follows from proposition 1 that any anonymous VNM social ordering $\succsim$ of $\mathbb{L}$ satisfying the weak Pareto principle with respect to a profile $\langle \succsim_i \rangle_{i=1}^n$ of identical selfish VNM preferences can be written, for lotteries p and q in $\mathbb{L}$, as:

$$p \succsim q \Leftrightarrow \sum_{i \in N} \sum_{j \in S} \pi^p_{ij} U_j(y^p_{ij}) \geq \sum_{i \in N} \sum_{j \in S} \pi^q_{ij} U_j(y^q_{ij}) \tag{5}$$

[4]The careful writting of this anonymity condition requires additional "multi-profiles" conditions on the social ordering. Details as to how can this be done are available in Coulhon and Mongin (1989) or Blackorby *et al.* (2005) (ch. 7).

The empirically implementable criteria proposed in this paper coincide with the ranking of lotteries in $\mathbb{L}$ that would be agreed upon by *all* social orderings that can be written as per (5) for some individual state dependant utility functions U_j (for $j \in S$) taken from some (reasonably large) class. To that extent, the considered criteria shall be referred to as "robust".

Yet, it is probably worth noticing that the legitimacy of formula (5) for comparing socially risky situations rides on *both* the "*ex ante*" view that ethical judgements on socially risky situations should result from the aggregation of individuals' preferences before the resolution of uncertainty *and* the assumption that the social ranking should satisfy VNM properties. The later requirement may be seen as particularly demanding in view of the well-known criticism that has been launched against it by Diamond (1967). Indeed, if one is indifferent between giving a kidney for sure to Bob and giving a kidney for sure to Ann, then, by virtue of VNM assumption, one must also be indifferent between giving the kidney for sure to either one of the two persons and basing the decision of who should get the kidney on the flip a coin. Since spontaneous intuition often tends to favour the flip of a coin in a situation like this, some are willing to endorse Roemer (1996)'s (p. 140) view that, with this example, "Diamond has presented a knockdown argument against the ethical attractiveness of" equation (5) as a basis for comparing socially risky situations. Yet others, such as Fleurbaey (2006), are less convinced by the devastating character of Diamond's critique. After all, since in the end only one person gets the kidney anyway, why should flipping a coin be considered better than giving the kidney for sure to either one of the two ?

Be it as it may, we base our notion of normative dominance on formula (5). Specifically, we define the notion of normative dominance with respect to a class of individual state dependant expected utility functions as follows.

Definition 1 (Normative dominance) *Socially risky situation p normatively dominates socially risky situation q for a class $\mathbb{U}$ of l state-dependant utility functions $U_j : \mathbb{R}_+ \to \mathbb{R}$ (for $j \in S$), denoted $p \succsim_{\mathbb{U}} q$, if for all combinations of l functions U_j in the class, one has:*

$$\sum_{i \in N} \sum_{j \in S} \pi_{ij}^p U_j(y_{ij}^p) \geq \sum_{i \in N} \sum_{j \in S} \pi_{ij}^q U_j(y_{ij}^q)$$

In order to define formally the class of state-dependant VNM utility functions that we consider, we introduce the following notation which eases the statement of the properties of functions that are *not* assumed to be everywhere differentiable. Specifically, if G is a function from a subset A of $\mathbb{R}$ to $\mathbb{R}$ and α is a number in A, we denote, for every strictly positive real number Δ such that $\alpha + \Delta \in A$ and by $G^{\Delta}(\alpha)$ the discrete (right-hand side) first derivative of G evaluated at α by:

$$G^{\Delta}(\alpha) = \frac{G(\alpha+\Delta) - G(\alpha)}{\Delta} \tag{6}$$

This definition can, of course, be applied recursively for different Δ so that, for instance, $G^{\Delta_1\Delta_2}(\alpha)$, interpreted to be the second order discrete derivative of G, is defined by:

$$G^{\Delta_1\Delta_2}(\alpha) = \frac{\frac{G(\alpha+\Delta_2+\Delta_1)-G(\alpha+\Delta_2)}{\Delta_1} - \left[\frac{G(\alpha+\Delta_1)-G(\alpha)}{\Delta_1}\right]}{\Delta_2}$$

With this notation, the two classes of state-dependant VNM utility functions considered in this paper are formally introduced as follows.

$$\begin{aligned}\mathbb{U}_1 = \{&U_j : \mathbb{R}_+ \to \mathbb{R} \text{ for } j = 1, ..., l : U_j^{\Delta}(\alpha) \geq U_{j+1}^{\Delta}(\alpha) \geq 0 \text{ and} \\ &U_j(\alpha) \leq U_{j+1}(\alpha) \text{ for all } \alpha,\ \Delta \in \mathbb{R}_+ \text{ and } j = 1, ..., l-1\ \}\end{aligned}$$

$$\begin{aligned}\mathbb{U}_2 = \mathbb{U}_1 \cap \{&U_j : \mathbb{R}_+ \to \mathbb{R} \text{ for } j = 1, ..., l : U_j^{\Delta_1\Delta_2}(\alpha) \leq U_{j+1}^{\Delta_1\Delta_2}(\alpha) \leq 0 \\ &\text{for all } \alpha,\ \Delta_1 \text{ and } \Delta_2 \in \mathbb{R}_+ \text{ and } j = 1, ..., l-1\}\end{aligned}$$

The class $\mathbb{U}_1$ contains all utility functions that are, in every state, *increasing in income* and that satisfy the additional requirements that, for a given income level,

1) the utility enjoyed weakly increases with the state and

2) the marginal utility income weakly decreases with the state.

The class $\mathbb{U}_2$ contains all functions belonging to $\mathbb{U}_1$ that satisfy the additional properties that:

1) the marginal utility of income is decreasing, in every state, with income and

2) the decrease in the marginal utility of income is weakly more important in bad states than in good ones.

As usual with dominance analysis, the larger is the class of utility functions over which unanimity is seek for, the harder it is to obtain firm conclusion but the more robust is the conclusion achieved. We now introduce easy-to-check implementable criteria that happen to coincide with normative dominance for each of these two classes of utility functions.

2.2 Implementable criteria

The *first* implementable criterion that is introduced is the *Sequential Expected Headcount Poverty* (SEHP) dominance criterion. It is formally defined as follows.

Definition 2 (Sequential Expected Headcount Poverty dominance) *For every socially risky situations p and $q \in \mathbb{L}$, p SEHP dominates q, denoted $p \succsim_{SEHP} q$ if, for every poverty line $t \in \mathbb{R}_+$ and every state $k \in S$,* *one has:*

$$\sum_{i\in N}\sum_{\{j:j\leq k\ ,\ y_{ij}^{p}\leq t\}}\pi_{ij}^{p}\leq\sum_{i\in N}\sum_{\{j:j\leq k\ ,\ y_{ij}^{q}\leq t\}}\pi_{ij}^{q} \tag{7}$$

In words, socially risky situation p dominates socially risky situation q for the SEHP criterion if, for every state and poverty line, the expected numbers of individuals who are *both* in a weakly worse state *and* poor is no greater in p than in q. As the SEHP criterion requires inequality (7) to hold for every poverty line, it implies, by choosing a large enough poverty line, that, for every state k, the expected number of individuals in states weakly worse than k be no greater in the dominating situation than in the dominated one. In the same spirit, since the SEHP criterion requires inequality (7) to holds for $k = l$, it implies the expected number of poor irrespective of the state to be no greater in the dominating situation than in the dominated one. Notice that requiring the *expected* number of poor irrespective of the state to be lower in the dominating situation is *not* equivalent to requiring the same relationship to hold for the *total* number of poor irrespective of the state.

The *second* implementable criterion is the analogue of SEHP dominance, but with *poverty gap*, rather than headcount poverty, used as a measure of poverty. We call it, for this reason, the Sequential Expected Poverty Gap (SEPG) criterion. In order to define this criterion, we denote by $P(t,a)$ the poverty gap of income a for the poverty line t defined by:

$$P(t,a) = \max[t-a,0] \tag{8}$$

This poverty gap is, as usual, interpreted to be the minimal amount of income required to get a person with income of a out of poverty when the poverty line is t. We then define the SEPG criterion as follows.

Definition 3 (Sequential Expected Poverty Gap dominance) *For every p and $q \in \mathbb{L}$, we say that p SEPG dominates q, denoted $p \succsim_{SEPG} q$ if, for every poverty line $t \in \mathbb{R}_+$ and every state $k \in S$, one has:*

$$\sum_{i=1}^{n}\sum_{j=1}^{k}\pi_{ij}^{p}P(t,y_{ij}^{p})\leq\sum_{i=1}^{n}\sum_{j=1}^{k}\pi_{ij}^{q}P(t,y_{ij}^{q}) \tag{9}$$

and, for every state k, it is the case that:

$$\sum_{i=1}^{n}\sum_{j=1}^{k}\pi_{ij}^{p}\leq\sum_{i=1}^{n}\sum_{j=1}^{k}\pi_{ij}^{q} \tag{10}$$

In words, socially risky situation p dominates socially risky situation q for the SEPG criterion if, for every poverty line and every state, the *expected amount of money* required to eliminate poverty *in all weakly worse* *states* is lower in p than in q (condition (9)) and if the expected amount of individuals who are in a weakly worse state is also weakly smaller in p than in q (condition (10)). It should be noticed that requirement (10) must be *added* to the definition of the criterion rather than being deduced, as was the case for the SEHP criterion, from the single inequality (9). As can be checked from the proof of theorem 2 below, the additional requirement (10) can be dispensed with if one assumes the existence of a sufficiently high income level for which an individual is indifferent between states. We must admit that we find this assumption rather implausible. After all, even arbitrarily rich individuals experience utility loss when they are raped.

It can also be noticed that the SEPG dominance criterion is formally equivalent to one proposed by Bazen and Moyes (2003) and Jenkins and Lambert (1993) as a generalization of the Atkinson and Bourguignon (1987) criterion for comparing distribution of incomes between households with differing needs. There is, indeed, a formal analogy between the problem of ranking distributions of incomes across households with different needs and that of comparing distribution of lotteries over state contingent income. The analogy consists in identifying a state with a need category (ordered from the more to the less needy), and interpreting the ratio of the sum of the individual probabilities of being in any state (need category) over the total number of individuals as the (marginal) probability of having an individual belonging to the need category. With this interpretation, the SEPG criterion is nothing else than the generalization of the Atkinson and Bourguignon (1987) criterion provided by Bazen and Moyes (2003) (if one includes in its definition inequality (10)) or by Jenkins and Lambert (1993) (if one does not include inequality (10)).

The next subsection provides normative foundation for each of these criteria by showing that it coincides with the ranking of socially risky situations that commands unanimity over all anonymous Paretian and VNM social rankings who assume that individual preferences can be represented by expected utility functions in one the two classes defined above.

2.3 Equivalence results

The first theorem establishes an equivalence between normative dominance for the class $\mathbb{U}_1$ and SEHP dominance. The proofs of all theorems have been relegated in Appendix 1.

Theorem 1 *Let p and q be two socially risky situations in $\mathbb{L}$. Then $p \succsim_{\mathbb{U}_1} q$ if and only if $p \succsim_{SEHP} q$.*

The next theorem establishes the equivalence between normative dominance over the class $\mathbb{U}_2$ and sequential expected poverty gap dominance.

Theorem 2 *Let p and q be two socially risky situations in $\mathbb{L}$. Then $p \succsim_{\mathbb{U}_2} q$ if and only if $p \succsim_{SEPG} q$.*

2.4 Risks with only pecuniary consequences

There are many instances where risks can be assumed to have only pecuniary consequences. Unemployment risk may be considered one of them if we abstract from the value of leisure or from the moral stigma that can be attached to unemployment status. If risks are perceived by individuals as having only pecuniary consequences, then individual i's ranking $\succsim_i$ of any two socially risky situation p and q writes:

$$p \succsim_i q \Leftrightarrow \sum_{j\in S} \pi_{ij}^p U(y_{ij}^p) \geq \sum_{j\in S} \pi_{ij}^p U(y_{ij}^q). \tag{11}$$

for some (state independant) utility function $U : \mathbb{R}_+ \to \mathbb{R}$. One can therefore defines as follows the notion of normative dominance that applies for such a situation.

Definition 4 *Socially risky situation p normatively dominates socially risky situation q for a class $\widehat{\mathbb{U}}$ of state independent utility functions $U : \mathbb{R}_+ \to \mathbb{R}$, denoted $p \succsim_{\widehat{\mathbb{U}}} q$, if for all functions U in the class, one has:*

$$\sum_{i\in N}\sum_{j\in S} \pi_{ij}^p U(y_{ij}^p) \geq \sum_{i\in N}\sum_{j\in S} \pi_{ij}^q U(y_{ij}^q)$$

There are naturally two classes of state independent utility functions that come to mind. The first one, referred to as $\widehat{\mathbb{U}}_1$, is the (large) class of all functions that are increasing in income. The second one, $\widehat{\mathbb{U}}_2$, consists in all functions in $\widehat{\mathbb{U}}_1$ that satisfy, in addition, the property of being concave with respect to income. We formally define these two classes as follows.

$\widehat{\mathbb{U}}_1 = \{U : \mathbb{R}_+ \to \mathbb{R} : U^{\Delta}(\alpha) \geq 0\}$

$\widehat{\mathbb{U}}_2 = \{U : \mathbb{R}_+ \to \mathbb{R} : U^{\Delta_1\Delta_2}(\alpha) \leq U^{\Delta_1\Delta_2}(\alpha) \leq 0$ for all Δ_1 and Δ_2 $\in \mathbb{R}_+\}$

It is not hard to identify the two implementable criteria that coincide with the rankings of socially risky situations agreed upon by all Paretian and VNM social planners who believe that individual VNM preferences can be defined by utility functions in $\widehat{\mathbb{U}}_1$ and $\widehat{\mathbb{U}}_2$ respectively.

The first criterion is Expected Headcount Poverty (EHP) dominance. It considers that a socially risky situation is better than another if the expected number of individuals who are poor in all states is no greater in the dominating distribution than in the dominated one. This criterion is clearly more discriminatory than SEHP dominance because requiring sequentially expected poverty to be lower in all states that are weakly worse than a given state implies requiring expected poverty to be lower in all states. This criterion is formally introduced as follows.

Definition 5 (Expected Headcount Poverty dominance) *For all socially risky situations p and $q \in \mathbb{L}$, p EHP dominates q, denoted $p \succsim_{EHP} q$ if, for every poverty line $t \in \mathbb{R}_+$, one has:*

$$\sum_{i \in N} \sum_{\{j:\, y_{ij}^p \leq t\}} \pi_{ij}^p \leq \sum_{i \in N} \sum_{\{j: y_{ij}^q \leq t\}} \pi_{ij}^q \tag{12}$$

Non surprisingly, the second criterion is Expected Poverty Gap (EPG) dominance. It works just like SEPG dominance but by requiring, less demandingly, that expected poverty gap be lower in the dominating distribution in the dominated one in all states rather than in all states worse than any given state. It is formally defined as follows.

Definition 6 (Expected Poverty Gap dominance) *For every p and $q \in \mathbb{L}$, we say that p EPG dominates q, denoted $p \succsim_{EPG} q$ if, for every poverty line $t \in \mathbb{R}$, one has:*

$$\sum_{i \in N} \sum_{j \in S} \pi_{ij}^p P(t, y_{ij}^p) \leq \sum_{i \in N} \sum_{j \in S} \pi_{ij}^q P(t, y_{ij}^q) \tag{13}$$

As can be noticed, EPG dominance does not require any additional condition - such as (10), on the probabilities of falling in the states. It only requires that the expected amount of money that would be necessary to eliminate poverty in all states be no greater in the dominating distribution than in the dominated one. Again, EPG dominance is implied by SEPG dominance but the converse implication does not hold. We now establish, in the two next theorems, that EHP and EPG dominance are equivalent to normative dominance over the class $\widehat{\mathbb{U}}_1$ and $\widehat{\mathbb{U}}_2$ respectively. We provide the proofs of these theorems in appendix 1 for the sake of completeness but the structure of these proofs mimic very closely those of theorems 1 and 2.

Theorem 3 *Let p and q be two socially risky situations in $\mathbb{L}$. Then $p \succsim_{\widehat{\mathbb{U}}_1} q$ if and only if $p \succsim_{EHP} q$.*

Theorem 4 *Let p and q be two socially risky situations in $\mathbb{L}$. Then $p \succsim_{\widehat{\mathbb{U}}_2} q$ if and only if $p \succsim_{EPG} q$.*

3 Empirical illustrations

We now illustrate how the criteria studied in this paper can generate interesting empirical normative conclusions. These illustrations are all made using sample data on different risks. In order to derive from these sample data conclusions that are valid for the populations represented by the samples, we perform statistical inference based on the Union-Intersection (UI) method as initiated by Bishop *et al.* (1989)[5]. The UI method supposes that we accept the hypothesis of dominance of a socially risky situation A over a socially risky situation B if none of the poverty inequalities that define the dominance criterion is significantly positive and at least one of the inequality is significantly negative. A brief description of this inference method is provided in appendix 2. All comparisons that are presented herein are performed at the 95% confidence level.

3.1 Evolution of risk of crime in India

It is well-known that India has experienced a period of spectacular economic growth in the last twenty years or so. While much researchers who have performed normative evaluation of the Indian growth experience on the sole basis of pecuniary considerations have concluded that this experience has been beneficial for India, there has been few studies that have looked at the impact of the Indian growth on the distribution of non-pecuniary attributes. In one of these studies, Gravel and Mukhopadhyay (2007) have considered, along with individual expenditures, three other non-pecuniary attributes measured at the level of the district of residence of the individuals: infant mortality, literacy rate, and probability of being the victim of a violent crime. They have conducted the analysis using Atkinson and Bourguignon (1982) multidimensional criteria and have concluded that, if one abstracts from crime, there has been a steady and robust increase in social welfare in India over the whole period 1987-2001 as recorded by the Atkinson and Bourguignon *first order dominance criterion.* However, when the probability of being the victim of a violent crime is added to the list of attributes, the conclusion holds only for the more ethically demanding *second order dominance criterion* of Atkinson and Bourguignon (1982), and is not valid for the whole period.

In Gravel and Mukhopadhyay (2007), no use is made of the fact that the distribution, between individuals, of probability of crime and expenditures can be viewed as a socially risky situation with two states - viz. being the victim of a criminal act or not. It is therefore of interest to see wether the normative appraisal of the evolution of risk of violent crime in India over the period is affected by accounting for this fact, in the line of the criteria proposed herein.

[5] See Howes (1994) for a critical appraisal of this inference methodology.

For this sake, we use the same data set as Gravel and Mukhopadhyay (2007). Data on households' consumption expenditure are taken from the 43rd (1987-1988), 52nd (1995-1996) and 58th rounds (2002) of the consumption expenditure surveys conducted by the National Sample Survey Organization (NSSO) of India. Individual consumption expenditures have been derived from household consumption expenditures using the Oxford equivalence scale and are in 2002 Rupees[6]. Consumption data have also been made comparable, to the extent possible, in terms of the reference period over which consumption expenditures are recollected by surveyed households (see e.g. Deaton and Drèze (2005) or Himanshu and Sen (2005) for in depth discussion of the problems raised by changes in the recall period used in the NSSO questionnaires during the period 1988-2002) At the all India level[7], the analysis is based on 131,511 individuals in 2002, 203,228 individuals in 1995-96 and 563,931 individuals in 1987-88.

We have assigned to each individual the violent crime rate of his or her district of residence, as provided by the NSSO data (there were 527 districts in India in 2002). Due to subdivisions in the district areas that have taken place in India over the 1981-2001 period, there are more districts in 2001 and 1991 than in 1981. In order to make the comparisons consistent, we have aggregated data for 1991 and 2001 to adhere to the original coarser 1981 districts partition. Violent crime rates (number of murders, attempted murders, and rapes per million individuals per district) have been obtained, for the same years as expenditure data, from the Indian National Crime Record Bureau. We have restricted our attention to the most violent and extreme form of crime to reduce the risk of trend biases due to the evolution of the reporting behavior of the victims of crimes (or their families). It is indeed well-known that crime reporting tends to grow with education and wealth. Our assumption is that, while present, this bias was less important for violent crimes, who tend to be reported to the police no matter what is the wealth or education level of the victim's family, than for robberies, burglaries, and other types of crimes.

Figures 1a and Ib show, respectively, the expected headcount poverty curves in the bad state and the expected headcount poverty curves irrespective of states (which coincide with standard headcount poverty curves here because individual consumption is the same in the two states for all individuals). From these pictures, one can see a clear ranking of the headcount poverty curves (from 1988 to 2002) and a dominance ranking of the expected headcount poverty curve in the bad state of 2002 over either 1988 or 1996. Yet the expected Headcount poverty curves in the bad state cross between 1996 and 1988, indicating a failure to reach clear cut conclusion as to the

[6] Price deflators are the Urban Non Manual Employees price index for urban data and Agricultural Labourers price index for rural ones. Comparisons or pooling between urban and rural data are performed using Deaton (2005) (table 17;3) ideal Fisher index.

[7] Jammu-Kashmir and North East states have been excluded from our study.

ranking of these two years. This impossibility to conclusively rank 1988 and 1996 is clear from the fact the average probability of being the victim of a violent crime in India has increased between 1988 and 1996.

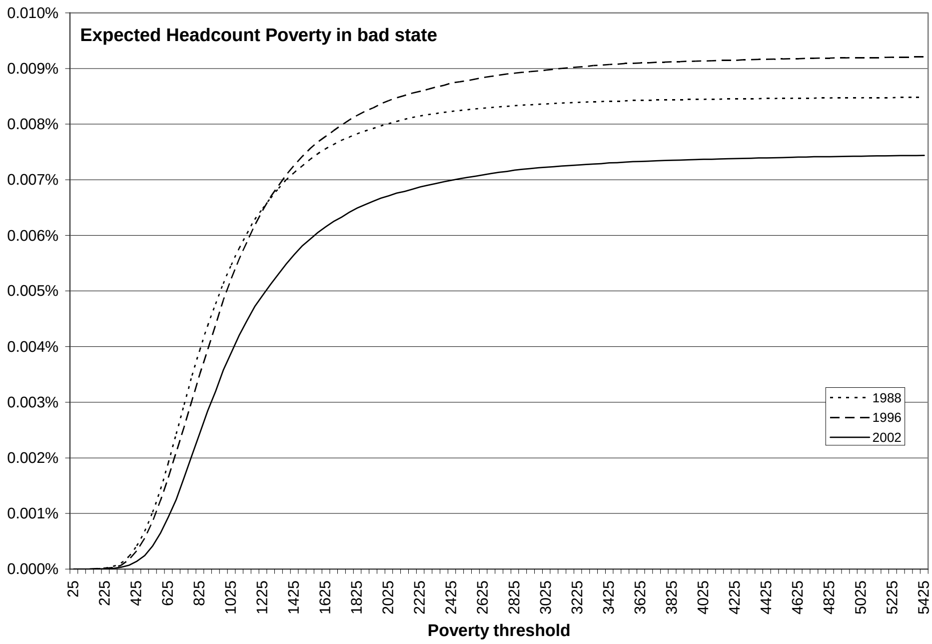

Figure 1a

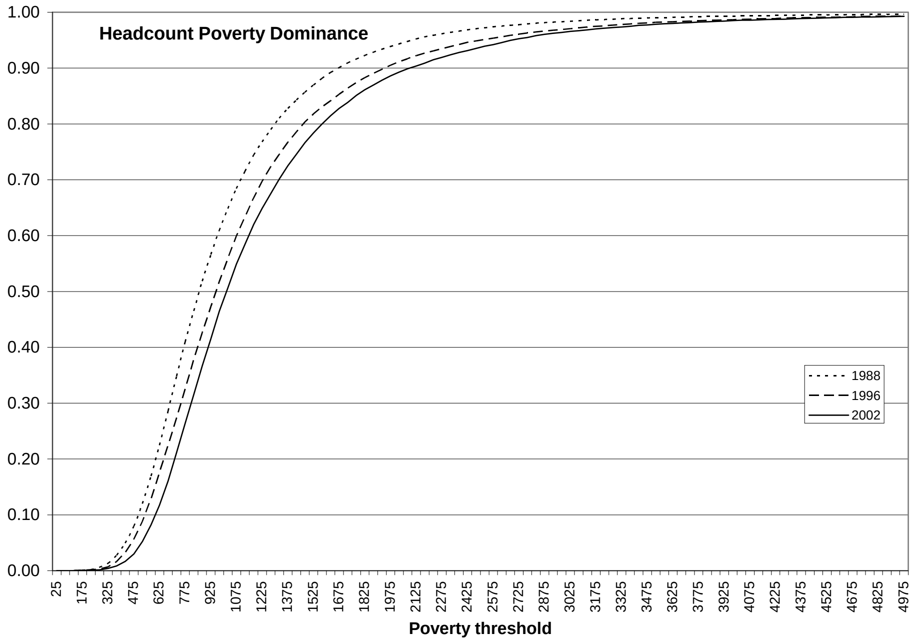

Figure 1b

This visual impression is confirmed by the statistical inference tests whose results are shown in Table 1 below.

	2002 vs 1996		**2002 vs 1988**	
Ranking	2002 $\succsim_{\mathbb{U}_1'}$ 1996		2002 $\succsim_{\mathbb{U}_1'}$ 1988	
T-statistics	Min	Max	Min	Max
SEHP condition (7)	-105.36	-2.21	-106.72	-2.72
Critical value SMM(92,∞): 3.449				

Table 1: Results of the comparisons with statistical inference

As is clear from the table, the distribution of risk of violent crime in 2002 dominates both that of 1996 or 1988 according to the SEHP dominance criterion. Yet, for the reason indicated above, no conclusion can be obtained for comparison of 1996 and 1988. This illustrates that the criterion proposed in this paper is quite useful to obtain robust normative conclusions. In Gravel and Mukhopadhyay (2007), no first order dominance could be obtained, and one had to resort to second order Atkinson and Bourguignon (1982) multidimensional dominance, and the rather demanding properties on the individual utility function that this criterion requires, to obtain the verdict. Here, we only need to accept that Indians have VNM preferences for risks that satisfy the rather mild property that the (positive) marginal utility of money is larger in the bad state than in the good one to reach the conclusion, provided of course that we accept to compare social risks by a Pareto inclusive and anonymous VNM social preference. Moreover, if we are willing to assume that Indian households do not value intrinsically the states in which they fall - a somewhat implausible assumption for crime - then the expected headcount poverty criterion enables us to rank conclusively all three periods.

3.2 Labor market related risks

In the last decade or so, there has been a growing concern for job security and a perception, by many workers in many countries, that their job was becoming increasingly unstable. These concerns have initiated a literature that attempt at empirically measuring the evolution of job insecurity in various western countries. For the United States, authors like Farber (2004), have suggested that the average job insecurity in the US has increased mildly in the late nineties while others, like Gottschalk and Moffit (1999), have shown no evidence of an increase in the probability of loosing one's job. Similarly, there has been several papers who have examined the evolution of the average risk of involuntary job loss in France. For instance, Givord and Maurin (2004) suggests that the probability of involuntary job loss has increased since the 1980s. It has also been noticed by Postel-Vinay (2003) that the increase in the risk of loosing one's job has been larger for low-seniority workers than for high-seniority ones. These studies have focused on the average probability of being unemployed and have not derived meaningful normative conclusion out of their analysis. The criteria examined in this paper are potentially useful for appraising the trends in individual exposures to (two-states) risk of unemployment.

We illustrate this by comparing the exposures to risks of unemployment of single adult members of the workforce between US and France. We focus on single adults to avoid, at this stage, normatively challenging issues that concern multi-individual households. We use for this purpose the French Labor Force Survey (LFS) and the US Current Population Survey-March Supplement (CPS-MS) for both 2003 and 2004. The LFS contains 50,524 respondents (employees and unemployed) among whom are there 6,953 single individuals without children. In the US CPS, there are 90,314 respondents (employees and unemployed) among whom 7,523 are single without children. In both data sets, the same individuals are observed in 2003 and 2004. The fact that some of them have experienced change in employment status between the two years enables us to assign to each individual in the sample a probability of being unemployed, an income if employed and a (substitution) income if unemployed.

The *first* element for defining involuntary unemployment risk is the probability of being involuntary unemployed in 2004. This probability means different things for different individual. For an individual observed *unemployed* in 2003, this probability is the probability of remaining unemployed in 2004. For an individual observed *employed* in 2003, it is the risk of loosing his or her job between the 2003 and 2004. We assign a probability to every individual by grouping them into homogeneous groups with respect to observable characteristics and by assigning to each individual of the group the same probability of unemployment. For individuals employed in 2003, this probability is the fraction of the individuals within the group who became unemployed in 2004. For unemployed individual in 2003, it corresponds to the fraction of individuals in the group that remained unemployed in 2004. There were 38 groups of employed individuals (formed with respect to the level of education, the activity sector, the age and the fact that they work in the private or the public sector) and 10 groups of unemployed (defined on the basis of education, unemployment seniority and gender). While the French LFS distinguishes between voluntary and involuntary unemployment, the CPS does not. Hence, we adjusted our estimated risks in the US by using the Displaced Workers Survey (DWS). The DWS is conducted in January only on the same sample of individuals used in the CPS. It asks workers whether or not they were involuntary displaced from a job at any time in the preceding three-year period. We use the DWS to estimate the fraction of unemployed individuals who have been involuntary put into that situation.

The *second* element needed to define a risk of involuntary unemployment is the labor income received by an employed individual and the (replacement) income received by the same individual when unemployed. The activity income of an individual observed employed in 2003 is simply the observed income of this individual provided by the data. If the individual is observed unemployed in 2003, we assign to this individual the monthly labor income he or she would have earned had he or she been employed. In order to assign this income to an unemployed person, we estimate a wage equation on the sample of employed individuals. We of course account for the possible selection bias that could arise from the fact that we assign to unemployed individuals a wage that has been estimated on a sample of employed households by using Heckman (1979)'s methodology. The independent variables used in the wage equation are seniority (dummy), occupations (6 dummies), industries (5 dummies), city size (10 dummies for France and 8 for United States), education level (6 dummies), age and age squared. We have performed the estimation separately for the samples of female and male singles. Gross activity income is then transformed for all individuals into disposable income by subtracting income taxes (net of possible income tax credit) and by adding welfare payments, if any. Finally, we assign to each individual a replacement income received in case of unemployment on the basis of the legislation in the two countries. This substitution income is principally made of unemployment benefits and/or social welfare payments. Unemployment benefits are function of the past activity income and the intensity of work (full/part time). Unemployment benefits are more generous in France (where they can last for one year) than in US (where they do not go beyond 26 weeks). In France the only welfare payment that is considered is the RMI (about 400 US a month) that works as a minimal income. As much welfare payments in the US are given to family with at least one children, we ignore these benefits in this study devoted to single adults. Other welfare payments like Housing benefits ("*Allocation Personnalisée au Logement*" and "*Allocation Logement*" in France, and Low-Rent Public Housing and Housing Choice Vouchers in the US) are also ignored since we do not have information on housing prices.

Summary statistics on probability estimates and average activity and replacement incomes are provided in table 2. All pecuniary figures are in US dollars, corrected for Purchasing Power Parity using OECD equivalence scales.

	France	United States
Probability of employment (%)	88.67	93.84
	(19.88)	(12.74)
Female	89.66	94.64
	(18.68)	(12.17)
Male	87.90	93.13
	(20.74)	(13.18)
<30 old	87.08	93.66
	(19.32)	(12.33)
>30 old	89.4	93.90
	(20.10)	(12.87)
Probability of remaining employed	95.99	95.51
	(3.97)	(9.45)
Probability of becoming employed	39.93	69.68
	(13.81)	(24.33)
Monthly income (PPP $)		
Mean income in employment	1,284	2,508
	(685.28)	(2428)
Mean replacement income	885	856
	(564.05)	(362.81)

Table 2: summary statistics, France USA

It is clear that the probability of being employed is both higher and more equally distributed amongst single workers in the US than in France. Notice that this seems to be true only for those individuals who were unemployed in 2003, as unemployment inertia is stronger in France than in the US. Among the employed individuals, there does not seem to be much difference between the probability of keeping one's job for one year in France (95.99 %) and in the US (95.51 %). This however seems to be specific to the population of single adults without children. The other estimations that we have done for the two populations suggest that, if we include the other members of the workforce, the probability of keeping one's job is also significantly higher in US than in France. Moreover France seems also to be more "unequal" than in the US in terms of the way it distributes the probability of keeping one's job across its single adults. The gap in average probability of good state is larger between women and men and between "old" (above thirty) and "young" workers in France than in the United States. We can also note that women seem to face, in both countries, a lower probability of being unemployed. This can be explained in part by the fact that there is a larger proportion of women working in the less risky public sector.

Can we now make more normatively meaningful comparisons between these two countries based on the criteria developed in this paper? In order to answer this question, we need first to order the two states of nature that define unemployment risks. There are really only three possibility here:

1) unemployment is intrinsically worse than employment,

2) unemployment is intrinscially better than employment and,

3) unemployment risks have only pecuniary consequences so that none of the two states is intrinsically better than the other.

The first possibility corresponds to a widespread view that unemployment has important adverse non-pecuniary consequences (social stigma, loss of self-esteem, etc.) that outweight the possible non-pecuniary benefit associated to the extra leisure time that it provides. The second possibility reflects the converse belief that the non pecuniary benefit of leisure time dominates the non-pecuniary cost of unemployment. The third possibility assumes that non-pecuniary negative and positive consequences of unemployement either cancel out each other or that their net effect is sufficiently small as compared to the pecuniary consequences that they can be neglected for all practical purposes. As we do not want to take any firm stance as to which of the three possibilities is the more likely, we apply the criteria to each of them in turn.

Figures 2a, 2b and 2c provide expected poverty gap curves in the unemployment state, in the employment state and in either state respectively for the total adult population of the two countries as well as the male and female subsamples. Recall from the definition of SEPG dominance that non-crossing of two curves in figures 2a and 2c is required if unemployment is considered to be a worse state than employment (first possibility). If the second possibility is considered, then non-crossing of two curves in figures 2b and 2c is required. Finally, if one assumes that unemployment risk has only pecuniary consequences, then the criterion of EPG dominance that applies in that case only requires non-crossing of the two curves in figure 2c.

Figure 2a.

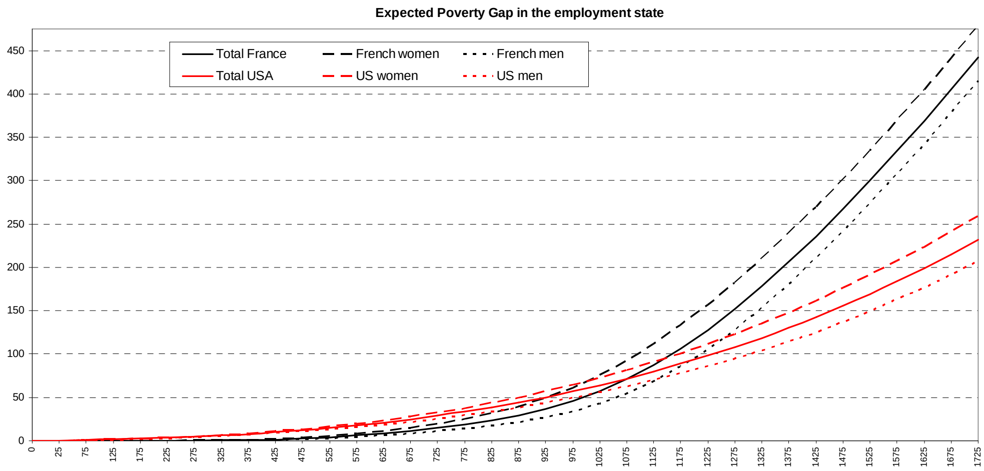

Figure 2b

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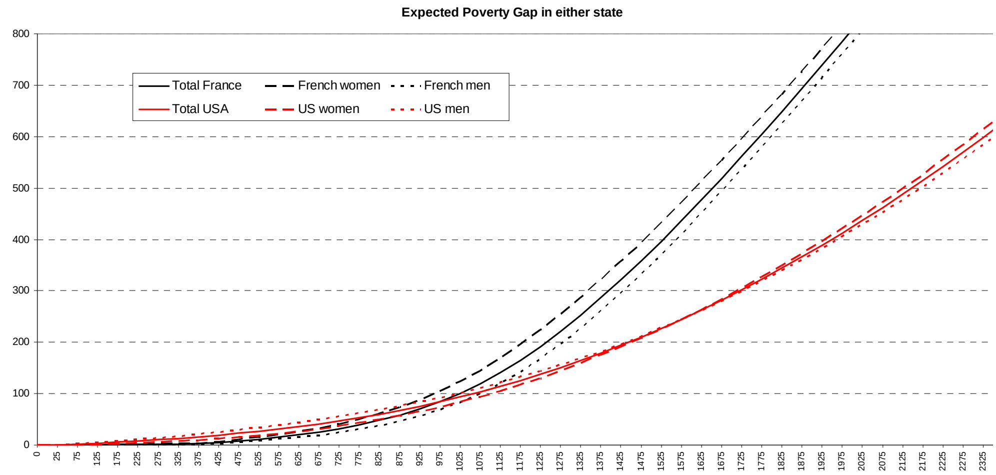

Figure 2c.

Since a crossing of any two curves on figure 2c is sufficient to block a dominance verdict for any of the three orderings of the two states, it seems clear that, irrespective of the population considered - male, female or the whole - there is no dominance between France and US. As appears clearly on figure 2c, the French curves lie below their US counterparts for very low levels of income (less than \$400 a month) while they lie above for the rest of the distribution. This crossing of the curves at \$400 happens of course because of the protection provided to extremely poor workers in France by the RMI, a protection that is not present in the US. Of course, only statistical testing can tell us if differences and crossing between curves are significant.

Interesting also are the comparisons of the male and female curves in each of the two countries. In the US, expected poverty gap irrespective of the state is higher for women than for men at the upper tail of the distribution but lower at the bottom part of it. On the other hand, if one concentrates on unemployed individuals, expected poverty gap is everywhere lower for women than for men. The situation is different in France where expected poverty gap irrespective of the state is lower for men than for women both in the unemployed states and irrespective of the states. Hence, while men seem to be robustly more protected against unemployment risk than women in France, no such dominance arises in the US. In this sense, it can be said that the men-women gap in terms of protection against unemployment risks is lower in the US than in France.

Table 3 shows the results of the comparisons, on the basis of the most discriminatory expected poverty gap criteria, of the above populations for all three orderings of the states based on statistical inference, rather than visual inspection of graphs. The table also shows the results of the

Comparison SEPG/EPG	**Ranking**	Min t	Max t	degree of freedom	critical t
FRANCE-US					
unemployment is bad	**?**	-18.07	63.32	$(396,\infty)$	3.95
employment is bad	?	-18.07	63.32	$(403,\infty)$	3.96
state independent	?	-18.07	63.32	$(201,\infty)$	3.66
FRANCE					
Fem.-Male, unemployment is bad	**?**	-3.42	9.57	$(158,\infty)$	3.59
Fem-Male, employment is bad	**Male**	-1.07	10.77	$(179,\infty)$	3.63
Fem-Male, state independent	**Male**	2.2	9.57	$(89,\infty)$	3.44
Young-old, unemployment is bad	**OLD**	4.67	12.38	$(157,\infty)$	3.59
Young-old, employment is bad	**?**	-2.35	12.38	$(179,\infty)$	3.63
Young-old, state independent	**OLD**	6.37	12.38	$(89,\infty)$	3.44
US					
Fem-male, unemployment is bad	?	-17.43	5.63	$(203,\infty)$	3.66
Fem-Male, employment is bad	?	-10.83	9.2	$(361,\infty)$	3.9
Fem-Male, state independent	?	-10.83	5.63	$(180,\infty)$	3.63
Old-Young, unemployment is bad	**OLD**	-1.96	14.65	$(203,\infty)$	3.66
Old-Young, employment is bad	**OLD**	3.36	14.65	$(361,\infty)$	3.9
Old-Young, state independent	**OLD**	1.7	14.65	$(180,\infty)$	3.63

Table 3a: SEHP or EHPcomparisons

comparisons, within each country, of the young and the old segment of the single adults sample (irrespective of gender), with 30 years old as the cut-off age.

Table 3 thus reinforces the impressions, provided by figures 2a-c, on the France-US diffences with respect to the way they expose their adult males and females to unemployment risks. As can be seen, except when unemployment is assumed to be the worse state, the exposure of French male single adults to unemployment risks is better than that of female. Moreover, the failure to achieve dominance of male over female in the case where unemployment is the worse state only comes from the fact that the average unemployment rate is lower in France among females than among males (remember that condition (10) must hold in order to have SEPG dominance). If one would be ready to assume that, at sufficiently high income level, there is no utility loss in being unemployed, then one could obtain the verdict that the exposure of French men to unemployment risks is better than that of French women even in the case where unemployment is taken to be the bad state.

Table 3 also reveals that, as can be expected, the old segment of the adult population is, in each country, better protected against unemployment risks than the young segment. As it turns out, this result holds even if one uses the more robust SEHP criterion. Notice that, in the case of France, the dominance of old adults over young ones does not hold if one assumes that employment is a worse state than unemployment at a given income level. The reason for this dominance failure comes, here again, from the failure to satisfy condition (10) (the probability of being employed is significantly higher in France among the old than than among the young).

4 Conclusion

This paper characterizes four robust criteria for comparing socially risky situations from a normative point of view, under two alternative assumptions on the nature of the risks faced by individuals. The criteria characterized are SEHP and SEPG dominance when risks are assumed to have both pecuniary and non-pecuniary consequences and are EHP and EPG dominance when risks are only assumed to have pecuniary consequences. These criteria are characterized as being ethically robust because each of them coincides with the ranking that command unanimous agreement among all VNM social ranking that are anonymous and Pareto inclusive with respect to a very wide class of identical individualistic VNM preferences. As briefly illustrated with empirical data, the dominance criteria are easy to use and capable of producing interesting conclusions. Among other things, as illustrated in the Indian example, they increase significantly the discriminatory power of more abstract multidimensional dominance criteria à la Atkinson and Bourguignon (1982). They are also useful in comparing different populations of individuals in terms of their exposure to risks of unemployment, where they showed, among other things, that the dominance of men over women seem to be less clear in the US than in France.

5 Appendix 1. Proofs of the theorems

5.1 Proof of theorem 1

For the first implication, assume that $p \succsim_{\mathbb{U}_1} q$. Then, the inequality:

$$\sum_{i\in N}\sum_{j\in S}\pi_{ij}^{p}U_j(y_{ij}^{p}) \geq \sum_{i\in N}\sum_{j\in S}\pi_{ij}^{q}U_j(y_{ij}^{q}) \tag{14}$$

holds for all combination of l functions $U_j : R_+ \to R$ (for $j \in S$) in U_1. Consider, for any $t \in R_+$ and $k \in \{1, ..., l\}$, the list of l functions $V_j^{tk} : R_+ \to R$ defined, for any $a \in R_+$ and $j \in \{1, ..., l\}$ by:

$$\begin{aligned} V_j^{tk}(a) &= -1 \text{ if } a \leq t \text{ and } j \leq k \\ &= 0 \text{ otherwise} \end{aligned}$$

It can be checked that any combination of l functions V_j^{kt} so defined belongs to U_1 for any $t \in R_+$.and $k \in \{1, ..., l\}$. Hence, inequality (14) holds for the combination of functions V_j^{kt} so that we have:

$$\begin{array}{rcl}
\sum_{i\in N}\sum_{j\in S}\pi_{ij}^p V_j^{kt}(y_{ij}^p) & \geq & \sum_{i\in N}\sum_{j\in S}\pi_{ij}^q V_j^{kt}(y_{ij}^q) \\
& \Leftrightarrow & \\
\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^p\leq t\}} -\pi_{ij}^p & \geq & \sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^q\leq t\}} -\pi_{ij}^q \\
& \Leftrightarrow & \\
\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^p\leq t\}} \pi_{ij}^p & \leq & \sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^q\leq t\}} \pi_{ij}^q
\end{array}$$

as required by (7).
For the other implication, we need to show that the fact of having (7) satisfied for every non-negative real number t and every $k \in S$ is sufficient for the inequality (14) to hold for all utility functions in U_1. For this sake, let $\overline{y}$ be a large enough positive real number and consider a subdivision of the interval $[0, \overline{y}]$ into r sub-intervals $[a_g, a_{g+1}]$ for $g = 0, ..., r-1$ such that:

$$\begin{array}{rcl}
a_0 & = & 0, \\
a_r & = & \overline{y}
\end{array}$$

and, for all $i \in N$ and $j \in S$, there exists g and $g' \in \{0, 1, ..., r\}$ such that $y_{ij}^p = a_g$ and $y_{ij}^q = a_{g'}$. We further assume, without loss of generality, that $a_{g+1} - a_g = \Delta > 0$ for all $g = 0, ..., r-1$. Analogously, we can consider a subdivision of the interval $[0, 1]$ into m sub-intervals $[\rho_h, \rho_{h+1}]$ for $h = 0, ..., m-1$ such that

$$\begin{array}{rcl}
\rho_0 & = & 0 \\
\rho_1 & = & 1
\end{array}$$

and, for all $i \in N$ and $j \in S$, there exists h and $h' \in \{0, 1, ..., m\}$ such that $\pi_{ij}^p = \rho_h$ and $\pi_{ij}^q = \rho_{h'}$. Considering these subdivisions of the intervals $[0, y]$ and $[0, 1]$, we can write (14) as:

$$\sum_{h=1}^{m}\sum_{j=1}^{l}\sum_{g=1}^{r}\Delta f_j(\rho_h, a_g)\rho_h U_j(a_g) \geq 0 \tag{15}$$

where, for $h = 1, ..., m$, $j = 1, ...l$ and $g = 1, ..., r$,

$$\Delta f_j(\rho_h, a_g) = \#\{i \in N : \pi_{ij}^p = \rho_h \ \&\ y_{ij}^p = a_g\} - \#\{i \in N : \pi_{ij}^q = \rho_h \ \&\ y_{ij}^q = a_g\} \tag{16}$$

(we of course allow for the possibility that the cardinality of either of the two sets that enters in that difference be zero).
We now proceed by decomposing the left hand side of (15) using Abel's identity (see

for instance (Fishburn and Vickson (1978); eq 2.49)). Doing first the decomposition with respect to the inner (g-indexed) summation operator yields:

$$\sum_{h=1}^{m}\sum_{j=1}^{l}[\sum_{g=1}^{r}\Delta f_j(\rho_h,a_g)\rho_h U_j(\overline{y})-\sum_{s=1}^{r-1}\sum_{g=1}^{s}\Delta f_j(\rho_h,a_g)\rho_h(\Delta U_j^{\Delta^s}(a_s))]\geq 0 \tag{17}$$

using the definition of a discrete derivative provided by (6). Decomposing (17) using Abel identity applied this time to the j-indexed sum operator yields:

$$\sum_{h=1}^{m}[\sum_{j=1}^{l}\sum_{g=1}^{r}\Delta f_j(\rho_h,a_g)\rho_h U_l(\overline{y})-\sum_{k=1}^{l-1}(\sum_{j=1}^{k}\sum_{g=1}^{r}\Delta f_j(\rho_h,a_g)\rho_h)(U_{k+1}(\overline{y})-U_k(\overline{y}))$$

$$-\sum_{j=1}^{l}\sum_{s=1}^{r-1}\sum_{g=1}^{s}\Delta f_j(\rho_h,a_g)\rho_h(\Delta U_l^{\Delta}(a_s))$$

$$+\sum_{k=1}^{l-1}\sum_{s=1}^{r-1}\sum_{j=1}^{k}\sum_{g=1}^{s}\Delta f_j(\rho_h,a_g)\rho_h(\Delta(U_{k+1}^{\Delta}(a_s)-U_k^{\Delta}(a_s)))]\geq 0 \tag{18}$$

Now, using (16), one can see that that, for every $s\in\{1,...,r\}$ and $k\in S$:

$$\sum_{h=1}^{m}\sum_{j=1}^{k}\sum_{g=1}^{s}\Delta f_j(\rho_h,a_g)\rho_h=\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^p\leq a_s\}}\pi_{ij}^p-\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^p\leq a_s\}}\pi_{ij}^q \tag{19}$$

Combining this with the fact that:

$$\sum_{h=1}^{m}\sum_{j=1}^{l}\sum_{g=1}^{r}\Delta f_j(\rho_h,a_g)\rho_h=\sum_{i\in N}\sum_{j\in S}\pi_{ij}^p-\sum_{i\in N}\sum_{j\in S}\pi_{ij}^q=n-n=0$$

we can write (18) as:

$$-\sum_{k=1}^{l-1}(\sum_{i\in N}\sum_{\{j:j\leq k\}}\pi_{ij}^p-\sum_{i\in N}\sum_{\{j:j\leq k\}}\pi_{ij}^q)(U_{k+1}(\overline{y})-U_k(\overline{y}))$$

$$-\sum_{s=1}^{r-1}(\sum_{i\in N}\sum_{\{j:y_{ij}^p\leq a_s\}}\pi_{ij}^p-\sum_{i\in N}\sum_{\{j:y_{ij}^q\leq a_s\}}\pi_{ij}^q)\Delta U_l^{\Delta}(a_s)$$

$$+\sum_{k=1}^{l-1}\sum_{s=1}^{r-1}(\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^p\leq a_s\}}\pi_{ij}^p-\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^q\leq a_s\}}\pi_{ij}^q)\Delta(U_{k+1}^{\Delta}(a_s)-U_k^{\Delta}(a_s))]\geq 0 \tag{20}$$

It is clear that having $\sum_{i\in N}\sum_{\{j:j\leq k\ \& y_{ij}^p\leq a_s\}}\pi_{ij}^p\leq\sum_{i\in N}\sum_{\{j:j\leq k\ \& y_{ij}^q\leq a_s\}}\pi_{ij}^q$ for all $k\in S$ and all real numbers t, and therefore all $a_s\in\{a_0,...,a_m\}$, is sufficient for inequality (20) to hold for all utility functions U_j in U_1.

5.2 Proof of theorem 2

As for the proof of the previous theorem, assume first that $p \succsim_{\mathbb{U}_2} q$ and, accordingly, that inequality (14) holds for all combination of l utility functions U_j:$\mathbb{R}_+ \rightarrow \mathbb{R}$ (for $j \in S$) in $\mathbb{U}_2$. Consider, for any $t \in \mathbb{R}_+$ and $k \in S$, the function $\widetilde{V}_j^{tk}$:$\mathbb{R}_+ \rightarrow \mathbb{R}$ defined, for any $a \in \mathbb{R}_+$ and $j \in S$, by:

$$\widetilde{V}_j^{tk}(a) = \min(a - t, 0) \text{ if } j \leq k$$

and:

$$\widetilde{V}_j^{tk}(a) = 0 \text{ otherwise}$$

For any $k \in S$ and $j \leq k$, the function $\widetilde{V}_j^{tk}$ is the "angle" function used in the classical proof of the Hardy-Littlewood-Polya theorem made by Berge (1959). The reader can verify that $\widetilde{V}_j^{tk}$ belongs to $\mathbb{U}_2$ (in particular $\widetilde{V}_j^{tk}$ is more concave than $\widetilde{V}_{j'}^{tk}$ if $j' \geq j$). For this reason the inequality:

$$\sum_{i \in N} \sum_{j \in S} \pi_{ij}^p \widetilde{V}_j^{tk}(y_{ij}^p) \geq \sum_{i \in N} \sum_{j \in S} \pi_{ij}^q \widetilde{V}_j^{tk}(y_{ij}^q) \tag{21}$$

holds for every t and k. Using the definition of the functions $\widetilde{V}_j^{tk}$, inequality (21) writes:

$$\begin{array}{ccc} \sum_{i \in N} \sum_{j \leq k} \pi_{ij}^p \min(y_{ij}^p - t, 0) & \geq & \sum_{i \in N} \sum_{j \leq k} \pi_{ij}^q \min(y_{ij}^q - t, 0) \\ & \Leftrightarrow & \\ \sum_{i \in N} \sum_{j \leq k} \pi_{ij}^p \max(t - y_{ij}^p, 0) & \leq & \sum_{i \in N} \sum_{j \leq k} \pi_{ij}^q \max(t - y_{ij}^q, 0) \end{array}$$

as required by condition (9) of SEPG dominance. To obtain condition (10) of SEPG dominance, we consider, for every $k \in S$, the functions V_j^k:
$\mathbb{R}_+ \rightarrow \mathbb{R}$ defined, for $j \in S$ and $a \in \mathbb{R}_+$, by:

$$\begin{array}{rcl} V_j^k(a) & = & -1 \text{ if } j \leq k \\ & = & 0 \text{ otherwise} \end{array}$$

These k-indexed functions clearly satisfy $V_{j+1}^k(a) \geq V_j^k(a)$ for every $a \in \mathbb{R}_+$ and $j = 1, ..., l-1$ and are all (trivially) increasing with respect to income for every j. It can be checked that these functions satisfy (very often trivially) the conditions imposed on the functions in $\mathbb{U}_2$. Hence, inequality (14) holds for any such functions V_j^k so that we have, for all $k \in S$:

$$\sum_{i \in N} \sum_{j \in S} \pi_{ij}^p V_j^k(y_{ij}^p) \geq \sum_{i \in N} \sum_{j \in S} \pi_{ij}^q V_j^k(y_{ij}^q)$$

or

$$\sum_{i=1}^{n}\sum_{j=1}^{k}-\pi_{ij}^{p} \geq \sum_{i=1}^{n}\sum_{j=1}^{k}-\pi_{ij}^{q}$$
$$\Leftrightarrow$$
$$\sum_{i=1}^{n}\sum_{j=1}^{k}\pi_{ij}^{p} \leq \sum_{i=1}^{n}\sum_{j=1}^{k}\pi_{ij}^{q}$$

as required by (10).
For the other implication, we proceed just as in the proof of theorem 1 by writing inequality (14) in the form of (15) and by doing the Abel decomposition of (15) until we reach condition (18). If we then proceed one step further and Abel decompose each term of (18) with respect to the inner (g-indexed) term, we obtain:

$$\begin{aligned}
&\sum_{h=1}^{m}[-\sum_{k=1}^{l-1}\sum_{j=1}^{k}\sum_{g=1}^{r}\Delta f_j(\rho_h,a_g)\rho_h(U_{k+1}(\overline{y})-U_k(\overline{y}))\\
&-\sum_{s=1}^{r-1}\sum_{g=1}^{s}\sum_{j=1}^{l}\Delta f_j(\rho_h,a_g)\rho_h\Delta U_l^{\Delta}(a_{r-1})\\
&+\sum_{v=1}^{r-2}\sum_{s=1}^{v}\sum_{g=1}^{s}\sum_{j=1}^{l}\Delta f_j(\rho_h,a_g)\rho_h\Delta^2 U_l^{\Delta\Delta}(a_v)\\
&+\sum_{k=1}^{l-1}\sum_{s=1}^{r-1}\sum_{j=1}^{k}\sum_{g=1}^{s}\Delta f_j(\rho_h,a_g)\rho_h\Delta(U_{k+1}^{\Delta^s}(a_{r-1})-U_k^{\Delta^s}(a_{r-1}))\\
&-\sum_{k=1}^{l-1}\sum_{v=1}^{r-2}(\sum_{s=1}^{v}\sum_{j=1}^{k}\sum_{g=1}^{s}\Delta f_j(\rho_h,a_g)\rho_h)\Delta^2(U_{k+1}^{\Delta\Delta}(a_v)-U_k^{\Delta\Delta}(a_v))] \geq 0
\end{aligned} \tag{22}$$

Noticing that:

$$\begin{aligned}
\sum_{h=1}^{m}\sum_{j=1}^{k}\sum_{s=1}^{v}\sum_{g=1}^{s}\Delta f_j(\rho_h,a_g)\rho_h\Delta &= \sum_{s=1}^{v}\Delta[\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^{p}\leq a_s\}}\pi_{ij}^{p}-\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^{q}\leq a_s\}}\pi_{ij}^{q}]\\
&= (a_1-a_0)(\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^{p}\leq a_1\}}\pi_{ij}^{p}-\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^{q}\leq a_1\}}\pi_{ij}^{q})\\
&\quad +(a_2-a_1)(\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^{p}\leq a_2\}}\pi_{ij}^{p}-\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^{q}\leq a_2\}}\pi_{ij}^{q})\\
&\quad +....\\
&\quad +(a_v-a_{v-1})(\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^{p}\leq a_v\}}\pi_{ij}^{p}-\sum_{i\in N}\sum_{\{j:j\leq k,\ y_{ij}^{p}\leq a_v\}}\pi_{ij}^{q})
\end{aligned}$$

$$
\begin{aligned}
&= a_v \sum_{i\in N} \sum_{\{j:j\le k,\ y^p_{ij}\le a_v\}} \pi^p_{ij} - \sum_{\{j:j\le k,\ y^p_{ij}\le a_v\}} \pi^p_{ij} y^p_{ij} \\
&\quad -(a_v \sum_{\{j:j\le k,\ y^q_{ij}\le a_v\}} \pi^q_{ij} - \sum_{\{j:j\le k,\ y^q_{ij}\le a_v\}} \pi^q_{ij} y^q_{ij})] \\
&= \sum_{i\in N} \sum_{\{j:j\le k,\ y^p_{ij}\le a_v\}} \pi^p_{ij}(a_v - y^p_{ij}) - \sum_{i\in N} \sum_{\{j:j\le k,\ y^q_{ij}\le a_v\}} \pi^q_{ij}(a_v - y^q_{ij}) \\
&= \sum_{i\in N}\sum_{j=1}^{k} \pi^p_{ij} P(a_v, y^p_{ij}) - \sum_{i\in N}\sum_{j=1}^{k} \pi^q_{ij} P(a_v, y^q_{ij})
\end{aligned}
$$

and remembering equation (19) in the proof of theorem 1, one can write (22) as:

$$
\begin{aligned}
&-\sum_{k=1}^{l-1}(\sum_{i\in N}\sum_{j=1}^{k} \pi^p_{ij} - \sum_{i\in N}\sum_{j=1}^{k} \pi^q_{ij})(U_{k+1}(\overline{y}) - U_k(\overline{y})) \\
&-(\sum_{i\in N}\sum_{j\in S} \pi^p_{ij} P(a_{r-1}, y^p_{ij}) - \sum_{i\in N}\sum_{j\in S} \pi^q_{ij} P(a_{r-1}, y^q_{ij}) U^{\Delta}_l(a_{r-1}) \\
&+\sum_{v=1}^{r-2}(\sum_{i\in N}\sum_{j\in S} \pi^p_{ij} P(a_v, y^p_{ij}) - \sum_{i\in N}\sum_{j\in S} \pi^q_{ij} P(a_v, y^q_{ij}))\Delta U^{\Delta\Delta}_l(a_v)
\end{aligned}
$$

$$
+\sum_{k=1}^{l-1}(\sum_{i\in N}\sum_{j=1}^{k} \pi^p_{ij} P(a_{r-1}, y^p_{ij}) - \sum_{i\in N}\sum_{j=1}^{k} \pi^q_{ij} P(a_{r-1}, y^q_{ij}))(U^{\Delta^s}_{k+1}(a_{r-1}) - U^{\Delta^s}_k(a_{r-1}))
$$

$$
\begin{aligned}
&-\sum_{k=1}^{l-1}\sum_{v=1}^{r-2}(\sum_{i\in N}\sum_{j=1}^{k} \pi^p_{ij} P(a_v, y^p_{ij}) \\
&-\sum_{i\in N}\sum_{j=1}^{k} \pi^q_{ij} P(a_v, y^q_{ij})\Delta(U^{\Delta\Delta}_{k+1}(a_v) - U^{\Delta\Delta}_k(a_v))] \ge 0
\end{aligned}
\tag{23}
$$

For any combination of functions U_j (for $j \in S$) belonging to $\mathbb{U}_2$, it is sufficient for (23) to hold to have, for all $k \in S$:

$$
\sum_{i\in N}\sum_{j=1}^{k} \pi^p_{ij} - \sum_{i\in N}\sum_{j=1}^{k} \pi^q_{ij} \le 0,
$$

and:

$$
\sum_{i\in N}\sum_{j=1}^{k} \pi^p_{ij} P(t, y^p_{ij}) - \sum_{i\in N}\sum_{j=1}^{k} \pi^q_{ij} P(t, y^q_{ij}) \le 0
$$

for all positive real numbers t, as required by SEPG dominance.

5.3 Proof of theorem 3.

For the first implication, assume that $p \succsim_{\widehat{\mathbb{U}}_1} q$. Then, the inequality:

$$\sum_{i \in N} \sum_{j \in S} \pi_{ij}^p U(y_{ij}^p) \geq \sum_{i \in N} \sum_{j \in S} \pi_{ij}^q U(y_{ij}^q) \tag{24}$$

holds for all functions $U : \mathbb{R}_+ \to \mathbb{R}$ in $\widehat{\mathbb{U}}_1$. Consider, for any $t \in \mathbb{R}_+$, the function $V^t : \mathbb{R}_+ \to \mathbb{R}$ defined, for every $a \in \mathbb{R}_+$ by:

$$\begin{aligned} V^t(a) &= -1 \text{ if } a \leq t \\ &= 0 \text{ otherwise} \end{aligned}$$

It is clear V^t is increasing for any t. Hence, inequality (24) holds for V^t so that we have:

$$\begin{aligned} \sum_{i \in N} \sum_{j \in S} \pi_{ij}^p V^t(y_{ij}^p) &\geq \sum_{i \in N} \sum_{j \in S} \pi_{ij}^q V^t(y_{ij}^q) \\ &\Leftrightarrow \\ \sum_{i \in N} \sum_{\{j : y_{ij}^p \leq t\}} -\pi_{ij}^p &\geq \sum_{i \in N} \sum_{\{j : y_{ij}^q \leq t\}} -\pi_{ij}^q \\ &\Leftrightarrow \\ \sum_{i \in N} \sum_{\{j : y_{ij}^p \leq t\}} \pi_{ij}^p &\leq \sum_{i \in N} \sum_{\{j : y_{ij}^q \leq t\}} \pi_{ij}^q \end{aligned}$$

as required by (12).

For the other implication, we need to show that the fact of having (12) satisfied for every non-negative real number t is sufficient for the inequality (24) to hold for all utility functions in $\widehat{\mathbb{U}}_1$. For this sake, we proceed just like for theorem 1 by letting $\overline{y}$ be a large enough positive real number and by considering a subdivision of the interval $[0, \overline{y}]$ into r sub-intervals $[a_g, a_{g+1}]$ for $g = 0, ..., r-1$ such that:

$$\begin{aligned} a_0 &= 0, \\ a_r &= \overline{y} \end{aligned}$$

and, for all $i \in N$ and $j \in S$, there exist g and $g' \in \{0, 1, ..., r\}$ such that $y_{ij}^p = a_g$ and $y_{ij}^q = a_{g'}$. We further assume, without loss of generality, that $a_{g+1} - a_g = \Delta > 0$ for all $g = 0, ..., r-1$. As in theorem 1 also, we keep the subdivision of the interval $[0, 1]$ into m sub-intervals $[\rho_h, \rho_{h+1}]$ for $h = 0, ..., m-1$ such that:

$$\begin{aligned} \rho_0 &= 0 \\ \rho_1 &= 1 \end{aligned}$$

and, for all $i \in N$ and $j \in S$, there exist h and $h' \in \{0, 1, ..., m\}$ such that $\pi_{ij}^p = \rho_h$ and $\pi_{ij}^q = \rho_{h'}$. We can now write (24) as:

$$\sum_{h=1}^{m} \sum_{j=1}^{l} \sum_{g=1}^{r} \Delta f_j(\rho_h, a_g) \rho_h U(a_g) \geq 0 \tag{25}$$

where Δf_j is defined as per (16) in the proof of theorem 1. We now proceed by decomposing the left hand side of (25) using Abel's identity. Doing first the decomposition with respect to the inner (g-indexed) summation operator yields:

$$\sum_{h=1}^{m}\sum_{j=1}^{l}\sum_{g=1}^{r}\Delta f_j(\rho_h,a_g)\rho_h U(\overline{y})-\sum_{h=1}^{m}\sum_{j=1}^{l}\sum_{s=1}^{r-1}\sum_{g=1}^{s}\Delta f_j(\rho_h,a_g)\rho_h \Delta U^{\Delta}(a_s)]\geq 0 \tag{26}$$

using again the definition of a discrete derivative provided by (6). Now, using (16), one can see that that, for every $s\in\{1,...,r\}$:

$$\sum_{h=1}^{m}\sum_{j=1}^{l}\sum_{g=1}^{s}\Delta f_j(\rho_h,a_g)\rho_h=\sum_{i\in N}\sum_{\{j:y_{ij}^p\leq a_s\}}\pi_{ij}^p-\sum_{i\in N}\sum_{\{j:y_{ij}^p\leq a_s\}}\pi_{ij}^q$$

Combining this with the fact that:

$$\sum_{h=1}^{m}\sum_{j=1}^{l}\sum_{g=1}^{r}\Delta f_j(\rho_h,a_g)\rho_h=\sum_{i\in N}\sum_{j\in S}\pi_{ij}^p-\sum_{i\in N}\sum_{j\in S}\pi_{ij}^q=n-n=0$$

It is clear that having $\sum_{i\in N}\sum_{\{j:y_{ij}^p\leq a_s\}}\pi_{ij}^p\leq\sum_{i\in N}\sum_{\{j:y_{ij}^q\leq a_s\}}\pi_{ij}^q$ for all real numbers t, and therefore all $a_s\in\{a_0,...,a_m\}$, is sufficient for inequality (26) to hold for all utility functions U in $\widehat{\mathbb{U}}_1$.

5.4 Proof of theorem 4

For the necessity part of the proof, assume that $p\succsim_{\widehat{\mathbb{U}}_2}q$ and, accordingly, that inequality (24) of the proof of theorem 3 holds for all utility functions U in $\widehat{\mathbb{U}}_2$. Consider, for any $t\in\mathbb{R}_+$, the (angle) function $\widetilde{V}^t:\mathbb{R}_+\to\mathbb{R}$ defined, for any $a\in\mathbb{R}_+$, by:

$$\widetilde{V}^t(a)=\min(a-t,0)$$

Since the function $\widetilde{V}^t$belongs to $\widehat{\mathbb{U}}_2$, the inequality:

$$\sum_{i\in N}\sum_{j\in S}\pi_{ij}^p\widetilde{V}^t(y_{ij}^p)\geq\sum_{i\in N}\sum_{j\in S}\pi_{ij}^q\widetilde{V}^t(y_{ij}^q)$$

holds for every positive t so that we have:

$$\begin{array}{ccc}\sum_{i\in N}\sum_{j\in S}\pi_{ij}^p\min(y_{ij}^p-t,0) & \geq & \sum_{i\in N}\sum_{j\in S}\pi_{ij}^q\min(y_{ij}^q-t,0)\\ & \Leftrightarrow & \\ \sum_{i\in N}\sum_{j\leq k}\pi_{ij}^p\max(t-y_{ij}^p,0) & \leq & \sum_{i\in N}\sum_{j\leq k}\pi_{ij}^p\max(t-y_{ij}^q,0)\end{array}$$

for all t, as required by the definition of EPG dominance.
For the other implication, we proceed again as in theorem 3 by writing inequality

(24) in the form of (25) and by doing the Abel decomposition of (25) until we reach condition (26). If we then proceed one step further and Abel decompose each term of (26)) with respect to the inner (g-indexed) term, we obtain (acknowledging that $\sum_{h=1}^{m}\sum_{j=1}^{l}\sum_{g=1}^{r}\Delta f_j(\rho_h, a_g)\rho_h = \sum_{i\in N}\sum_{j\in S}\pi_{ij}^{p} - \sum_{i\in N}\sum_{j\in S}\pi_{ij}^{q} = 0)$:

$$
\begin{aligned}
&-\sum_{h=1}^{m}\sum_{j=1}^{l}[\sum_{s=1}^{r-1}\sum_{g=1}^{s}\Delta f_j(\rho_h, a_g)\rho_h \Delta U^{\Delta}(a_{r-1})] + \\
&+\sum_{v=1}^{r-2}\sum_{s=1}^{v}\sum_{g=1}^{s}\Delta f_j(\rho_h, a_g)\rho_h \Delta^2 U^{\Delta\Delta}(a_v)] \geq 0
\end{aligned}
\tag{27}
$$

As in the proof of theorem 2, it can be noticed that:

$$
\begin{aligned}
\sum_{h=1}^{m}\sum_{j=1}^{l}\sum_{s=1}^{v}\sum_{g=1}^{s}\Delta f_j(\rho_h, a_g)\rho_h \Delta &= \sum_{s=1}^{v}\Delta[\sum_{i\in N}\sum_{\{j:y_{ij}^{p}\leq a_s\}}\pi_{ij}^{p} - \sum_{i\in N}\sum_{\{j:y_{ij}^{q}\leq a_s\}}\pi_{ij}^{q}] \\
&= (a_1 - a_0)(\sum_{i\in N}\sum_{\{j:y_{ij}^{p}\leq a_1\}}\pi_{ij}^{p} - \sum_{i\in N}\sum_{\{j:y_{ij}^{q}\leq a_1\}}\pi_{ij}^{q}) \\
&\quad +(a_2 - a_1)(\sum_{i\in N}\sum_{\{j:y_{ij}^{p}\leq a_2\}}\pi_{ij}^{p} - \sum_{i\in N}\sum_{\{j:y_{ij}^{q}\leq a_2\}}\pi_{ij}^{q}) \\
&\quad +.... \\
&\quad +(a_v - a_{v-1})(\sum_{i\in N}\sum_{\{j:y_{ij}^{p}\leq a_v\}}\pi_{ij}^{p} - \sum_{i\in N}\sum_{\{j:y_{ij}^{p}\leq a_v\}}\pi_{ij}^{q}) \\
&= a_v\sum_{i\in N}\sum_{\{j:y_{ij}^{p}\leq a_v\}}\pi_{ij}^{p} - \sum_{\{j:y_{ij}^{p}\leq a_v\}}\pi_{ij}^{p}y_{ij}^{p} \\
&\quad -(a_v\sum_{\{j:y_{ij}^{q}\leq a_v\}}\pi_{ij}^{q} - \sum_{\{j:y_{ij}^{q}\leq a_v\}}\pi_{ij}^{q}y_{ij}^{q})] \\
&= \sum_{i\in N}\sum_{\{j:y_{ij}^{p}\leq a_v\}}\pi_{ij}^{p}(a_v - y_{ij}^{p}) - \sum_{i\in N}\sum_{\{j:y_{ij}^{q}\leq a_v\}}\pi_{ij}^{q}(a_v - y_{ij}^{q}) \\
&= \sum_{i\in N}\sum_{j\in S}\pi_{ij}^{p}P(a_v, y_{ij}^{p}) - \sum_{i\in N}\sum_{j\in S}\pi_{ij}^{q}P(a_v, y_{ij}^{q})
\end{aligned}
$$

For this reason one can write (27) as:

$$
\begin{aligned}
&-[\sum_{i\in N}\sum_{j\in S}\pi_{ij}^{p}P(a_v, y_{ij}^{p}) - \sum_{i\in N}\sum_{j\in S}\pi_{ij}^{q}P(a_v, y_{ij}^{q})]U^{\Delta}(a_{r-1})] + \\
&+\sum_{v=1}^{r-2}[\sum_{i\in N}\sum_{j\in S}\pi_{ij}^{p}P(a_v, y_{ij}^{p}) - \sum_{i\in N}\sum_{j\in S}\pi_{ij}^{q}P(a_v, y_{ij}^{q})]\Delta U^{\Delta\Delta}(a_v)] \geq 0
\end{aligned}
$$

Quite clearly, for any function U in $\widehat{\mathbb{U}}_2$, it is sufficient for this inequality to hold to have:

$$\sum_{i\in N}\sum_{j\in S}\pi_{ij}^{p}P(t,y_{ij}^{p})-\sum_{i\in N}\sum_{j\in S}\pi_{ij}^{q}P(t,y_{ij}^{q})\leq 0$$

for all positive real number t, as required by SEPG dominance.

6 Appendix 2. Testing methodology

We briefly describe here the UI inference methodology advocated by Bishop *et al.* (1989) or Bishop and Formby (1999)) that is used in this paper.

Consider two sampled risky situations p and q and denote, for $r=p,q$ and $k=1,...,l$, by $\widehat{D}_k^r(t)$ the expected number of poor in states weakly worse than k in risky situation r for the poverty line t. This is defined by:

$$\widehat{D}_k^r(t)=\frac{1}{n_r}\sum_{i=1}^{n_p}\sum_{\{j:j\leq k,\ y_{ij}^r\leq t\}}\pi_{ij}^r$$

where n_r is the sample size of situation r. Given observations of the numbers $\widehat{D}_k^p(t)-\widehat{D}_k^q(t)$ for $k=1,...,l$ all relevant poverty lines t (the number of which can be restricted to be the number of distinct levels of income observed in the two samples in the various states), the question is:

When we can infer from these observations that the inequalities (7) that define SEHP dominance are satisfied for the population of individuals that are represented by the samples ?

Consider a grid of m relevant poverty lines $(t_1;...;t_m)$ and define, for every poverty line t, the statistics:

$$\widehat{T}_h^k(t)=\frac{\widehat{D}_k^p(t_h)-\widehat{D}_k^q(t_h)}{\left(\widehat{\omega}_k^p(t_h)/n_p-\widehat{\omega}_k^q(t_h)/n_q\right)^{1/2}} \tag{28}$$

where, for $h=1,...,k$, and $r=p,q$, the number $\widehat{\omega}_k^r(t_h)$ is the estimation of the asymptotic variance of $\widehat{D}_k^r(t_h)$. Using the same reasoning as Davidson and Duclos (2000) and Duclos *et al.* (2006) based on the law of large numbers and the central limit theorem, these estimations of the asymptotic variance are defined by:

$$\widehat{\omega}_k^r(t_h)=\frac{1}{n_r}\sum_{i=1}^{n_r}[\sum_{\{j:j\leq k,y_{ij}^r\leq t_h\}}\pi_{ij}^r]^2-[\widehat{D}_k^r(t_h)]^2]$$

We can provide similar numbers for the SEPG criterion. Indeed, we can denote by $\tilde{D}_k^r(t)$ the estimated expected poverty gap in all states weakly worse than k in the sample r (for $r=p,q$) that is defined by:

$$\tilde{D}_k^r(t)=\frac{1}{n_r}\sum_{i=1}^{n_p}\sum_{j=1}^{k}\pi_{ij}^r\max(t-y_{ij}^p)$$

We can also define the statistics $\widetilde{T}_h^k(t)$ analogous to that defined in (28) but based instead on the sample estimates $\tilde{\omega}_k^r(t_h)$ of the asymptotic variance of $\tilde{D}_k^r(t_h)\ _k)$ defined by:

$$\tilde{\omega}_k^r(t_h) = \frac{1}{n_p}\sum_{i=1}^{n}[\sum_{j=1}^{k}\pi_{ij}^r \max(t_h - y_{ij}^r)]^2 - [\tilde{D}_k^r(t_h)]^2$$

The UI inference rule says that we infer a SEHP (resp SEPG) dominance of p over q if none of the poverty inequalities that define the criterion is statistically positive and at least one is statistically negative. Formally, the UI rule says that:

- If $\max_{h,k}\{\widehat{T}_h^k(t_h)\}\ . < C_\alpha$ and $\min_{h,k}\{\widehat{T}_h^k\} < -C_\alpha$ (resp.$\max_{h,k}\{\widetilde{T}_h^k(t_h)\} < C_\alpha$ and $\min_{h,k}\{\widetilde{T}_h^k\} < -C_\alpha$ we infer that p SEHP (resp. SEPG) dominates q.
- If $\min_{h,k}\{\widehat{T}_h^k\} > -C_\alpha$ and $\max_{h,k}\{\widehat{T}_h^k(t_h)\} > C_\alpha$, (resp. $\min_{h,k}\{\widetilde{T}_h^k\} > -C_\alpha$ and $\max_{h,k}\{\widetilde{T}_h^k(t_h)\})$ we infer that q SEHP (resp. SEPG) dominates p.
- If $\min_{h,k}\{\widehat{T}_h^k\} > -C_\alpha$ and $\max_{h,k}\{\widehat{T}_h^k(t_h)\} < C_\alpha$, (resp. $\min_{h,k}\{\widetilde{T}_h^k\} > -C_\alpha$ and $\max_{h,k}\{\widetilde{T}_h^k(t_h)\} < C_\alpha)$, we infer that p and q are indifferent. for the SEHP (resp. SEPG) criterion
- we infer that p and q are not-comparable for the relevant criterion otherwise.

where C_α is the critical value for a level of significance of α determined from the *Studentized Maximum Modulus* distribution provided by Stoline and Ury (1979). The degree of freedom used is lm and corresponds to the number of equalities that we want to test simultaneously (m inequalities in each of the l states).

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